Chapter 8: Heat Transfer

1.) Basic Concepts and Modes of Heat Transfer
Heat Transfer may be defined as the transmission of energy from one region to another as a result of temperature gradient, and is governed by the second law of thermodynamics which dictates that free flow of heat is possible only from a body of higher temperature to that at a lower temperature. There are three basic modes of heat transfer.

a. Conduction: Conduction is the transfer of heat between two bodies or two parts of the same body in physical contact with it, through molecules without appreciable displacement of molecules. In solids, as copper wire, the energy transfer arises because atoms at a higher temperature vibrate more excitedly; hence they transfer energy to neighboring atoms. In metals, the free electrons also contribute to the heat conduction process. In liquids or gas, the molecules are also mobile, and energy is conducted by molecular collisions.

b. Convection: Convection is the transfer of heat within a fluid by the mass movement of fluids. When a temperature difference produces a density difference, which results in mass movement of the fluids and thus causes convection heat transfer, the process is called free or natural convection. When the mass movement is caused by an external device like pump, fan etc, and then the process is called forced convection.

c. Radiation: Radiation is the transfer of heat through space or matter by means of electromagnetic waves or photons. All bodies radiate heat, so a transfer of heat by radiation occurs because hot body emits more heat than it receives and a cold body receives more heat than it emits. Heat transfer through radiation requires no medium for propagation and will pass through a vacuum.

2.) Conduction: Conduction Rate Equation and Heat Transfer Coefficient

2.1 Fourier’s Law of Conduction
It states that the rate of flow of heat through a single homogeneous solid along one dimensional is directly proportional to the area of the section and temperature gradient along the length of the path of heat flow. Mathematically,

\[ \dot{Q} \propto A \frac{dT}{dx} \]

\[ \dot{Q} = -KA\frac{dT}{dx} \]

Where, \( K \) = constant of proportionally or thermal conductivity of the body. This eq(1) is also called as conduction rate equation. (-) negative sign indicates that heat naturally flow from high temperature to low temperature.

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Thermal Conductivity is the property of material which determines how much heat can be conducted through the material and is given by the rate of heat transfer through a body of unit area and unit thickness on a unit temperature difference.

2.2 One-dimensional heat transfer through slab:

\[ Q = -kA \frac{dT}{dx} = -kA \frac{(T_2 - T_1)}{x} \]

\[ q = \frac{Q}{A} = k \frac{T_1 - T_2}{x} \]

\[ \text{eq}(2) \]

**Electrical Analogy for thermal resistance:**

According to Fourier’s law, the heat flow rate is due to temperature difference which is very similar to current flow due to potential difference.

\[ Q = \frac{T_1 - T_2}{x} = \frac{T_1 - T_2}{R_{th}} \]

\[ \text{eq}(3) \]

\[ I = \frac{E_1 - E_2}{R} \]

\[ \text{eq}(4) \]

The heat flow rate equation (3) is quite like to ohm’s law equation (4). Therefore, here \( R_{th} = \frac{x}{kA} \) for slab, is known as thermal resistance. The resistance to flow of heat transfer from conduction is known as Thermal resistance. This is inversely proportional to the thermal conductivity (K) of the solid. The unit for the thermal resistance is °C/W.

2.3 One-dimensional heat transfer through Composite Wall (Series)

Figure below shows a system consisting of two layers of different materials placed in series so as to make a composite plane structure.
The temperature gradients in the two materials are also shown, and the heat flow may be written as

\[ Q = k_1 A \frac{T_1 - T'}{x_1} \]
\[ T_1 - T' = Q \left( \frac{x_1}{k_1 A} \right) \] \hspace{1cm} \text{eq}(5)\]

\[ Q = k_2 A \frac{T' - T_2}{x_2} \]
\[ T' - T_2 = Q \left( \frac{x_2}{k_2 A} \right) \] \hspace{1cm} \text{eq}(6)\]

From above equations,

\[ T_1 - T_2 = (T_1 - T') + (T' - T_2) \]
\[ = Q \left( \frac{x_1}{k_1 A} \right) + Q \left( \frac{x_2}{k_2 A} \right) \]
\[ = Q \left( \frac{x_1}{k_1 A} + \frac{x_2}{k_2 A} \right) \]

So,

\[ Q = \frac{(T_1 - T_2)}{\left( \frac{x_1}{k_1 A} + \frac{x_2}{k_2 A} \right)} = \frac{(T_1 - T_2)}{R_1 + R_2} \] \hspace{1cm} \text{eq}(7)\]

In eq(7), thermal resistances are added like the addition of resistance in series of electric circuit.

### 2.4 One-dimensional heat transfer through Composite Wall (Parallel)

Figure below shows a system consisting of two layers of different materials placed in parallel so as to make a composite plane structure. Here, heat flow rate Q divides into the individual walls as Q₁ and Q₂.
The temperature gradients in the two materials are also shown, and the heat flow rate through individual walls may be written as

\[ Q_1 = k_1 A_1 \frac{T_1 - T_2}{x} \]
\[ Q_2 = k_2 A_2 \frac{T_1 - T_2}{x} \]

So the net heat transfer would be,

\[ Q = Q_1 + Q_2 \]
\[ = \frac{T_1 - T_2}{x} \left( k_1 A_1 + k_2 A_2 \right) \]
\[ = \frac{T_1 - T_2}{x} \left( \frac{1}{R_1 + \frac{1}{R_2}} \right) \]
\[ = \frac{T_1 - T_2}{R_1 R_2 \left( R_1 + R_2 \right)} \]

In eq(8), thermal resistances are added like the addition of resistances in parallel of electric circuit.

2.5 Radial Heat Conduction through Tubes (Cylinder)

Let us assume the inside and outside surface of the cylinder are maintained at temperatures \( T_1 \) and \( T_2 \) respectively \( (T_1 > T_2) \). Let us assume that heat is flowing, under steady state, only in the radial direction, and there is no heat conduction along the length of the cylinder i.e. longitudinal direction. Take a small annular section with thickness \( dr \) at a radius \( r \).

Area of small annular section \( A = 2\pi r L \),

The heat transfer rate through the small annular section of thickness \( dr \) is given by,
\[ Q = -kA \frac{dT}{dr} \]

\[ Q = -k \cdot 2 \pi r L \frac{dT}{dr} \]

Where, \( L \) is the length of the cylinder.

Here, \( R_{th} = \left( \ln \left( \frac{R_2}{R_1} \right) \right) \left( \frac{1}{2 \pi k L} \right) \), this is thermal resistance for cylinder having outer radius \( R_2 \) and inner radius \( R_1 \), thermal conductivity \( K \) and length \( L \).

3.) Convection:

The appropriate rate equation for the convective heat transfer between a surface and an adjacent fluid is prescribed by *Newton's Law of cooling*:

*Newton’s Law of cooling* is defined as heat flow rate is directly proportional to area exposed to heat transfer and temperature difference between the surface and fluid temperatures.

\[ \dot{Q} \propto A(T_w - T_a) \]
The transmission of heat per unit time from a surface by convection is given by
\[ \dot{Q} = hA(T_w - T_x) \] ..........................eq(9)

\( \dot{Q} \) = rate of quantity of convective heat transferred
h = coefficient of convective heat transfer
A = area of surface exposed to heat transfer
\( T_w \) = temperature of surface
\( T_x \) = temperature of fluid

Consider the heated plate shown in figure. The temperature of the plate is \( T_w \), and the temperature of the fluid \( T_x \). The velocity of the flow will appear as shown, being zero at the plate as a result of viscous action. Since the velocity of the fluid layer at the wall will be zero, the heat must be transferred only by conduction at that point. The temperature gradient is dependent on the rate at which the fluid carries away the heat. A high velocity produces a large temperature gradient.

The coefficient of convective heat transfer ‘h’ may be defined as the rate of heat transmitted for a unit temperature difference between the fluid and unit area of surface.

The value of ‘h’ depends on the fluids, their velocity and temperature.

\[ Q = hA(T_w - T_x) \]
\[ = \frac{(T_w - T_x)}{hA} \] ..........................eq(10)

Here, \( R_m = \frac{1}{hA} \) is the convective resistance for convective medium.

3.1 Overall heat transfer coefficient:
Consider the plane wall having thermal conductivity \( K \) and thickness ‘x’ shown exposed to a hot fluid A with convective heat transfer coefficient \( h_1 \) and temperature \( T_A \) on one side and a cooler fluid B with convective heat transfer coefficient \( h_2 \) and temperature \( T_B \) on other side.
The heat transferred is expressed by:

$$Q = h_1 A (T_A - T_1) = \frac{kA}{x} (T_1 - T_2) = h_2 A (T_2 - T_B)$$

So,

$$Q = \frac{T_A - T_B}{1 + \frac{x}{h_1 A} + \frac{1}{k A} + \frac{1}{h_2 A}}$$

$$Q = \frac{A (T_A - T_B)}{1 + \frac{x}{h_1 A} + \frac{1}{h_2 A}}$$

...eq(11)

The overall heat transfer by combined conduction and convection is frequently expressed in terms of an overall heat transfer coefficient $U$, and defined by the relation;

$$Q = UA(T_A - T_B)$$

...eq(12)

Comparing eq(11) and eq(12) we get the overall heat transfer coefficient $U$ as;

$$U = \frac{1}{1 + \frac{x}{h_1 A} + \frac{1}{k A} + \frac{1}{h_2 A}}$$

is known as overall heat transfer coefficient.

$$U = \frac{1}{R_{value}}$$

Similarly, for a hollow cylinder exposed to a convection environment on its inner and outer surfaces. Let the $T_A$ and $T_B$ be the inner and outer fluid temperatures, and $h_i$ and $A_i$ are the inner fluid convective heat transfer coefficient and inner surface area of cylinder. Likewise, $h_o$ and $A_o$ are for outer fluid convective heat transfer coefficient and outer surface area.

$$Q = \frac{(T_A - T_B)}{1 + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} + \frac{1}{h_o A_o}}$$
3.2 Introduction to Boundary Layer

Consider a heated wall surface temperature $T_s$ over which a fluid with undisturbed velocity $U_\infty$ and temperature $T_\infty$. The particles of fluid in intimate contact with the plate tend to adhere to it and a region of variable velocity builds up between the plate surface and the free fluid stream as indicated in Figure.

The fluid velocity decreases as it approaches the solid surface, reaching to zero in the fluid layer immediately next to the surface. This thin layer of stagnated fluid has been called the hydrodynamic boundary layer. The quantity of heat transferred is highly dependent upon the fluid motion within this boundary layer, being determined chiefly by the thickness of the layer. The boundary layer thickness $\delta$ is arbitrarily defined as the distance $y$ from the plate surface at which the velocity approaches 99% of free stream velocity.

**Reynolds Number** (Re): Reynolds number is a ratio of the inertia force to the viscous force. It indicates the relative importance of inertial and viscous effects in a fluid motion. At low Reynolds number, the viscous effects dominate and the fluid motion is laminar. At high Reynolds number, the inertial effects lead to turbulent flow and the associated turbulence level dominates the momentum and energy flux.

$$Re = \frac{\rho V^2}{\mu V/L} = \frac{\rho V L}{\mu}$$

**Prandtl Number** (Pr): It indicates the relative ability of the fluid to diffuse momentum and internal energy by molecular mechanisms.

$$Pr = \frac{\mu C_p}{K} = \frac{\rho V C_p}{K}$$

**Nusselt Number** (Nu): Nusselt Number establishes the relation between convective film coefficient $h$ and the thermal conductivity of the fluid $K$ and a significant length parameter length $l$ of the physical system.

$$Nu = \frac{hL}{K}$$

**Stanton Number** (St): It is the ratio of heat transfer coefficient to the flow of heat per unit temperature rise due to the velocity of the fluid.

$$St = \frac{h}{\rho V C_p} = \frac{hL/K}{(\rho V L/\mu)(\mu C_p/K)} = \frac{Nu}{Re.Pr}$$

**Forced Convection:**

a. Laminar Flow over a Flat Plate ($Re < 4 \times 10^5$)

   Local heat transfer Coefficient:
   $$Nu_s = 0.332(Pr)^{1/3}(Re)^{1/2}$$

   Average heat transfer Coefficient:
   $$Nu = 0.664(Pr)^{1/3}(Re)^{1/2}$$

b. Turbulent Flow over a Flat Plate ($Re > 5 \times 10^5$)

   Local heat transfer Coefficient:
   $$Nu_s = 0.0292(Pr)^{0.33}(Re)^{0.8}$$

   Average heat transfer Coefficient:
   $$Nu = 0.036(Pr)^{0.33}(Re)^{0.8}$$
4) Radiation:
The maximum rate of radiation that can be emitted from a surface at an absolute temperature $T$ (in K or R) is given by the Stefan–Boltzmann law as the maximum emissive power of a black body is directly proportional to fourth power of its absolute temperature and area.

$$Q_{\text{max}} \propto AT^4$$

$$Q_{\text{max}} = \sigma AT^4 \quad \text{eq}(13)$$

Here, $\sigma$ is the Stefan Boltzmann constant $= 5.669 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$

The idealized surface that emits radiation at this maximum rate is called a blackbody, and the radiation emitted by a blackbody is called blackbody radiation. The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as;

$$Q = \varepsilon \sigma AT^4 \quad \text{eq}(14)$$

where $\varepsilon$ is the emissivity of the surface. The property emissivity, $\varepsilon$, whose value is in the range $0 \leq \varepsilon \leq 1$, is a measure of how closely a surface approximates a blackbody for which $\varepsilon = 1$.

The radiant heat-exchange between two grey bodies at temperature $T_1$ and $T_2$ depends on how the two bodies view each other (Shape Factor $F_{1-2}$) and their emissivities, and is given by

$$Q = \sigma AF_{1-2}(T_1^4 - T_2^4) \quad \text{eq}(15)$$

4.1 Radiation Properties

When radiant energy strikes a material surface, part of the radiation is reflected, part is absorbed and part is transmitted. We define reflectivity ($\rho$) as the fraction reflected, the absorptivity ($\alpha$) as the fraction absorbed, and the transmissivity ($\tau$) as the fraction transmitted.

where $E$ is the radiation energy incident on the surface, and $E_{\text{abs}}$, $E_{\text{ref}}$, and $E_{\text{tr}}$ are the absorbed, reflected, and transmitted portions of it, respectively. The first law of thermodynamics requires that the sum of the absorbed, reflected, and transmitted radiation energy be equal to the incident radiation. That is,

$$E_{\text{abs}} + E_{\text{ref}} + E_{\text{tr}} = E$$

Dividing each term of this relation by $E$ yields Thus,

$$\rho + \alpha + \tau = 1 \quad \text{eq}(16)$$

Most solid bodies don’t transmit thermal radiation i.e. opaque, so that for many applied problems the transmissivity may be taken as zero. So,
The emissive power ($E$) of a body is defined as the energy emitted by the body per unit area and per unit time.

### 4.2 Black and Grey Bodies

A **black body** is defined as the body which absorbs all the incident radiation.

\[ \alpha = 1, \quad \rho = \tau = 0 \]

\[ E_b = \sigma T^4 \]

Real bodies are not ‘black’ and radiate less energy than the black body. To account this emissivity ($\varepsilon$) is defined in terms of emissive powers of real body and the black body, both evaluated at same temperature. Thus,

\[ \varepsilon = \frac{E}{E_b} \]

The emissive power of a body to the emissive power of a black body at the same temperature is equal to the absorptivity of body. So, $\varepsilon = \alpha$. The emissivity of a material varies with temperature and the wavelength of radiation.

Radiation of all frequencies is emitted from a hot surface. The emissive power $E$, therefore has contributions at all frequencies, and we write $E = \int_0^\alpha E_\lambda d\lambda$.

The term **black** and **grey** do not necessarily refer to the color of the body; they merely describe its effectiveness as a radiator. A black body is a perfect radiator; a grey body is not perfect radiator.